COMPLEX PERMITTIVITY MEASUREMENTS OF EXTREMELY LOW LOSS DIELECTRIC MATERIALS USING WHISPERING GALLERY MODES

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Abstract

Whispering-gallery modes are used for very accurate complex permittivity measurements of both isotropic and uniaxially anisotropic dielectric materials. A mode-matching technique is used to find the relationship between the complex permittivity, resonant frequency, and the dimensions of a resonant structure. The total uncertainty in permittivity is smaller than 0.05 percent and is limited principally by uncertainty in sample dimensions.

1 Theory and Experiments

Braginsky, Ilchenko, and Bagdassarov have described use of whispering gallery modes for dielectric loss tangent measurements of very low loss materials [1]. We present application of the whispering-gallery mode technique [2] for accurate measurements of the real permittivity and temperature coefficients of permittivity for both isotropic and uniaxially anisotropic dielectric materials. A radial mode-matching technique [3], [4] is used to find the relationship between the permittivity, sample dimensions and the resonant frequencies. To find two permittivity tensor components of an axially oriented cylindrical dielectric resonator, we solve two non-linear equations,

$$\begin{aligned} F_1(f^{(H)}, \epsilon_\perp, \epsilon_\parallel) &= 0 , \\ F_2(f^{(E)}, \epsilon_\perp, \epsilon_\parallel) &= 0 , \end{aligned} \qquad (1)$$

where $f^{(H)}$ and $f^{(E)}$ are measured resonant frequencies for a quasi-TE (H) and a quasi-TM (E) whispering gallery mode, and $\epsilon_\perp, \epsilon_\parallel$ are the real parts of the permittivity tensor components perpendicular and parallel to the anisotropy axis. The eigenvalue equations (1), represented by F_1 and F_2, are equations with respect to two permittivity tensor components in the form of a determinant of a square matrix. The determinant results from application of the mode-matching technique for symmetric and antisymmetric whispering gallery mode families. The size of the matrix depends on the number of terms taken into account in the field expansion series. Once permittivities are evaluated, dielectric loss tangents can be computed as solutions to the following linear equations,

$$\begin{aligned} Q_{(E)}^{-1} &= p_{e\perp}^{(E)} \tan\delta_\perp + p_{e\parallel}^{(E)} \tan\delta_\parallel + R_S/G^{(E)} , \\ Q_{(H)}^{-1} &= p_{e\perp}^{(H)} \tan\delta_\perp + p_{e\parallel}^{(H)} \tan\delta_\parallel + R_S/G^{(H)} , \end{aligned} \qquad (2)$$

where $\tan\delta_\perp, \tan\delta_\parallel$ are the dielectric loss tangents perpendicular and parallel to the anisotropy axis, $p_{e\perp}^{(H)}, p_{e\parallel}^{(H)}, p_{e\perp}^{(E)}, p_{e\parallel}^{(E)}$ are the electric energy filling factors perpendicular (subscript $\perp$) and parallel (subscript $||$) to the anisotropy axis of the resonant structure for quasi-TM whispering gallery modes (superscript (E)) and quasi-TE whispering gallery modes (superscript (H)), and $G^{(E)}$ and $G^{(H)}$ are the appropriate geometric factors.

To determine temperature coefficients of permittivity, we must know the temperature coefficients of resonant frequencies and expansion of the sample. Provided that the shield is sufficiently far from the dielectric sample, temperature coefficients of permittivity parallel, $\alpha_{\epsilon\|}$, and perpendicular, $\alpha_{\epsilon\perp}$, to the anisotropy axis can be found as solutions to the two linear equations,

$$\begin{aligned}\frac{1}{2}p_{e\perp}^{(H)}\alpha_{\epsilon\perp}+\frac{1}{2}p_{e\|}^{(H)}\alpha_{\epsilon\|} &= -\frac{1}{f^{(H)}}\frac{\partial f^{(H)}}{\partial T}-p_D^{(H)}\alpha_D \\ &\quad -p_L^{(H)}\alpha_L\ , \\ \frac{1}{2}p_{e\perp}^{(E)}\alpha_{\epsilon\perp}+\frac{1}{2}p_{e\|}^{(E)}\alpha_{\epsilon\|} &= -\frac{1}{f^{(E)}}\frac{\partial f^{(E)}}{\partial T}-p_D^{(E)}\alpha_D \\ &\quad -p_L^{(E)}\alpha_L\ ,\end{aligned} \tag{3}$$

where T denotes temperature, α_L and α_D are coefficients of linear expansion parallel and perpendicular to the anisotropy (cylinder) axis and

$$p_D^{(H)}=\mid\frac{\partial f^{(H)}}{\partial D}\mid\frac{D}{f^{(H)}}\ , \qquad p_D^{(E)}=\mid\frac{\partial f^{(E)}}{\partial D}\mid\frac{D}{f^{(E)}}\ ,$$
$$p_L^{(H)}=\mid\frac{\partial f^{(H)}}{\partial L}\mid\frac{L}{f^{(H)}}\ , \qquad p_L^{(E)}=\mid\frac{\partial f^{(H)}}{\partial L}\mid\frac{L}{f^{(E)}}\ ,$$

where D and L are the diameter and height of the dielectric resonator (Fig. 1). To find the temperature coefficients of permittivity, the resonant frequencies for a pair of modes must be measured as a function of temperature and coefficients of linear expansion must be known within the same temperature range. Experiments were performed for a sapphire resonator enclosed in a cavity (Fig. 1a) at cryogenic temperatures and for a low loss dielectric ceramic specimen without a shield at room and elevated temperatures. Measurement and computational results for the sapphire resonator are given in Table 1. Resonant frequency computations were performed for permittivities determined from the measured frequencies shown in bold in Table 1. The letter N used in mode designation describes antisymmetric modes having an equatorial symmetry plane at the center of the sample corresponding to a electric wall. The letter S designates symmetric modes whose symmetry plane corresponds to a magnetic wall. The numbers following the letters N and S represent subsequent modes that are ordered according to increasing resonant frequency belonging to either symmetric or antisymmetric mode families. For our sapphire resonator modes designated by N1 belong to the dominant quasi-TM family, whereas the modes designated by S2 belong to the dominant quasi-TE family. Table 1 shows that agreement between measured and computed resonant frequencies of a few whispering gallery modes

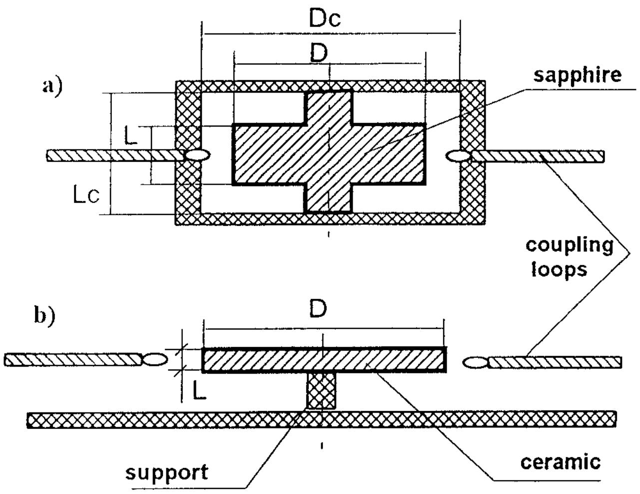

Figure 1: Whispering-gallery mode resonant structures used in laboratory experiments: (a) sapphire resonator, (b) dielectric ceramic resonator.

is within 0.01 percent. To determine the dielectric loss tangents we measured the unloaded Q-factors for the modes having large azimuthal mode numbers. In Table 2 computational results of the electric energy filling factors, geometrical factors, and measurement results of the unloaded Q-factors for the same sapphire rod are given. The electric energy filling factors show that both N1 and S2 mode families can be treated as quasi-TM and quasi-TE modes, respectively. Numerical data on geometric factor computations (column 5 of Table 2) demonstrate that, for large azimuthal mode numbers, conductor losses for whispering-gallery modes are smaller than dielectric losses and in most cases can be neglected. In this case, the dielectric loss tangents can be approximately evaluated as reciprocals of Q's for quasi-TM and quasi-TE modes. In summary, although conductor losses can be taken into account, they are negligible in practice. Dielectric loss tangent values are then most accurately determined, since no data on geometric factors and surface resistance are necessary.

To determine temperature coefficients of the permittivity tensor, we measured the resonant frequencies versus temperature for a pair of modes. One of the modes was a quasi-TM (N1, m=12) and the other quasi-TE (S2, m=10). Results of the evaluation of temperature coefficients of permittivity are shown in Fig. 2. Temperature coefficients of permittivity for sapphire exhibit anisotropy, just as do permittivity, dielectric loss tangents, and linear expansion coefficients. At temperatures between 4 and 70 K, the thermal coefficients of permittivity and expansion be-

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Table 1: Results of computations and measurements of the resonant frequencies (in GHz) at 77 K on sapphire rod with 49.9894 mm diameter and 30.008 mm height in a shield 80 mm diameter and 50 mm height. Permittivity components used in computations: $\epsilon_\perp = 9.2747, \epsilon_\| = 11.3532$.

m	N1 (comp.)	N1 (exp.)	S2 (comp.)	S2 (exp.)
8				
9			8.39804	8.39849
10			9.09276	9.09355
11	8.32828	8.32812	9.78712	9.78540
12	8.94870	**8.94870**	10.47400	**10.47400**
13	9.56728	9.56732	11.15975	11.16065
14	10.18407	10.18417	11.84238	11.84300
15	10.79920	10.79933		

Table 2: Electric energy filling factors, geometrical factors, and Q-factors (measured) for a few quasi-TM (N1) and quasi-TE (S2) modes of the sapphire resonator.

	m	$p_{e\perp}$	$p_{e\|}$	G (Ω)	Q (77 K)
N1	11	0.0470	0.9341	6.77×10^6	3.5×10^7
	12	0.0402	0.9423	1.63×10^7	3.7×10^7
	13	0.0350	0.9488	3.92×10^7	4.7×10^7
	14	0.0303	0.9538	9.42×10^7	4.2×10^7
S2	10	0.9548	0.0103	6.04×10^6	1.7×10^7
	11	0.9607	0.0064	1.60×10^7	1.9×10^7
	12	0.9620	0.0081	4.20×10^7	1.7×10^7
	13	0.9585	0.0106	1.10×10^8	1.7×10^7

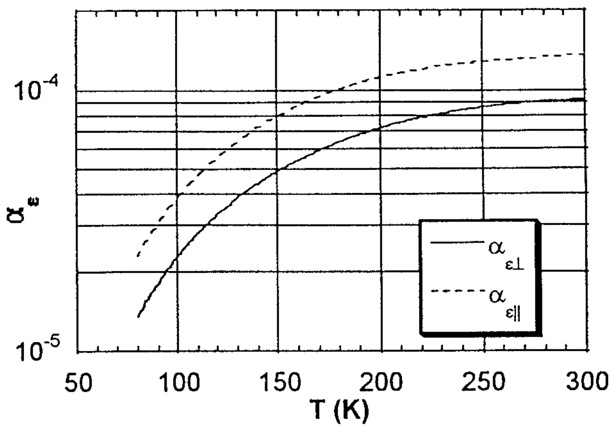

Figure 2: Thermal coefficients of permittivity for sapphire evaluated from measurements of whispering gallery mode resonant frequencies versus temperature.

come very small, approximately varying with temperature according to the power law T^4. the temperature coefficients of permittivity are difficult to determine at these temperatures, since the temperature dependence of the resonant frequencies can vary with the type and amount of atomic paramagnetic impurities [5].

Measurements of isotropic materials are easier to perform than anisotropic materials since only one mode needs identification to determine the real permittivity. Thus one eigenvalue equation is used instead of two. Measurement results and computations of the resonant frequencies of a low loss ceramic material at room temperature are given in Table 3. A relative permittivity equal to 36.55 (uncertainty of 0.3 percent) of this ceramic was initially measured at room temperature using a specimen as a $TE_{01\delta}$ mode dielectric resonator. The real permittivity was then evaluated from the data on one of the dominant quasi-TE whispering gallery modes (S1, m=8). Again, as for sapphire, good agreement between measured and computed resonant frequencies is apparent. Uncertainty analysis has shown that absolute uncertainty of permittivity is about 0.1 percent and depends on dimensional uncertainty of the sample. Since the measurements of the frequencies were performed over an octave frequency band, the whispering-gallery mode technique can be considered as a relatively broadband frequency method for permittivity measurements.

2 Summary

Use of whispering-gallery modes in dielectric resonators is one of the most accurate ways to determine complex permittivity and temperature coefficient of ultra-low loss isotropic and uniaxially anisotropic dielectric materials. Conductor losses can be made negligibly small, even for extremely low loss materials. This makes the whispering-gallery mode technique the most accurate and unique for measurements of extremely small dielectric loss tangents. With the utilization of several modes, complex permittivity measurements may be performed over a relatively broad frequency spectrum.

One of the most difficult aspects of measurements using whispering- gallery modes is proper mode identification. Mode identification is based on computations of resonant frequencies for initially determined permittivity values; when these initial permittivities

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Table 3: Measurement and computational results for resonant frequencies (in GHz) on dielectric disk 59.70 mm in diameter and 5.00 mm height. Permittivity used in computations: ϵ=36.550.

m	S1 (comp.)	S1 (exp.)	S2 (comp.)	S2 (exp.)
5	3.35622	3.35564	4.27180	4.27187
6	3.65909	3.65889	4.58974	4.59016
7	3.95471	3.95468	4.90088	4.90128
8	4.24489	**4.24489**		
9	4.53091	4.53109		
10	4.81371	4.81400		
11	5.09397	5.09459		

differ significantly from actual values, proper mode identification becomes uncertain. Despite this difficulty, the whispering-gallery mode technique may be conveniently applied at frequencies as high as 100 GHz [6].

3 References

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